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Design of PID, IMC-PI, and LQI Controllers for Infant Incubator Temperature Regulation

Salah I. Sawan¹, Aya H. Alsaid², *Ahmed J. Abougarair³

Department of Electrical and Electronic Engineering, Faculty of
Engineering, University of Tripoli, Tripoli, Libya^{1,2,3}

*Corresponding author: a.abougarair@uot.edu.ly

3- <https://orcid.org/0000-0001-9738-4888>

Abstract

This paper presents a comparative simulation study of three feedback-control strategies for regulating air temperature in a laboratory-scale infant incubator. A first-order thermal model is obtained from a lumped energy balance and used to design a practical proportional-integral-derivative controller with derivative filtering, an internal-model-control-based proportional- integral controller, and a linear-quadratic-integral controller. The simulations include actuator saturation, a heat-loss disturbance beginning at 5 s, plant-parameter variation, and measurement noise. Performance is evaluated by rise time, settling time, overshoot, integral squared error, integral time-weighted absolute error, and total variation of the heater command. Under nominal conditions, the LQI controller produces the fastest response and the lowest tracking indices, while the filtered PID provides a less aggressive practical compromise. The IMC-PI controller has the smoothest and most noise-insensitive behavior but is slower. The results provide a transparent controller-selection framework for an educational incubator-control design; they are not a clinical certification or a substitute for safety testing on medical equipment.

Keywords: Infant incubator, Temperature regulation, PID, IMC, LQI, Disturbance rejection, Robustness.

تصميم متحكمات PID و IMC-PI و LQI لتنظيم درجة حرارة حاضنة الأطفال

صلاح ابراهيم صوان¹، اية حسين السيد²، أحمد جابر أبوجير³

قسم الهندسة الكهربائية والإلكترونية - جامعة طرابلس - طرابلس - ليبيا^{1,2,3}

الملخص

تقدم هذه الدراسة محاكاة مقارنة لثلاثة طرق للتحكم في درجة حرارة الهواء داخل حاضنة أطفال. تم اشتقاق نموذج حراري بسيط من الدرجة الأولى باستخدام موازنة الطاقة، وتم على أساسه تصميم ثلاث متحكمات: متحكم PID مع فلتر تفاضلي، ومتحكم PI قائم على نظرية النموذج الداخلي، ومتحكم LQI الأمثل. تم اختبار هذه المتحكمات في ظل ظروف تحاكي الواقع العملي مثل تشيع سخان، واضطراب فقدان الحرارة، وتغير معاملات النظام، ووجود ضوضاء في قياس الحرارة. أظهرت النتائج أن متحكم LQI هو الأسرع والأدق في الظروف المثالية، بينما يقدم متحكم PID حلاً وسطاً عملياً، ويتميز متحكم IMC-PI ببساطة الأداء ومقاومته للضوضاء على حساب سرعة الاستجابة. تهدف هذه النتائج إلى توفير إرشادات واضحة لاختيار المتحكم المناسب في التطبيقات التعليمية، مع التأكيد على أن هذه النتائج محاكاة وليست بديلاً عن الاعتماد السريري أو اختبارات السلامة على الأجهزة الطبية الفعلية.

الكلمات المفتاحية: حاضنة الأطفال، تنظيم درجة الحرارة، PID، IMC، LQI، رفض الاضطرابات، المتانة.

Introduction

Infant incubators are controlled thermal environments intended to keep the air surrounding a newborn near a prescribed temperature. A practical controller must track the commanded temperature, reject heat loss when an access door is opened, and avoid rapid heater switching. Recent incubator studies have used on-off, PI, PID, fuzzy, and hybrid fuzzy- PID control approaches, which shows that the control problem remains relevant even when the hardware is small and low cost [1-3]. The selected control law must therefore

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balance tracking speed, actuator effort, sensitivity to sensor noise, and ease of implementation.

The temperature loop is especially suitable for a control- design term paper because its energy-balance model is interpretable, the actuator is naturally bounded by the available heater power, and disturbance scenarios have a direct physical meaning. The model used in this work represents a laboratory-scale chamber rather than a specific commercial incubator. Consequently, all results are simulation results and should not be interpreted as evidence of compliance with medical-device standards.

A conventional PID controller is simple to implement and remains a dominant industrial feedback-controller structure. However, derivative action can amplify sensor noise, and a nominally fast tuning can produce unnecessary heater activity. IMC-based tuning provides a direct way to trade speed against robustness through a single filter parameter [4]. Optimal state-feedback methods such as LQR can deliver fast responses when a state model is available, and integral augmentation removes steady-state error for constant references and disturbances [5,6].

All simulation results, controller designs, and performance evaluations presented in this paper were obtained using MATLAB. The software was employed for:

- Modeling the first-order thermal system of the infant incubator.
- Implementing the three controller structures: filtered PID, IMC-PI, and LQI.
- Conducting time-domain simulations under nominal conditions, disturbance rejection, plant-parameter variations, and measurement noise scenarios.
- Computing all performance indices, including rise time, settling time, overshoot, ISE, ITAE, and total variation (TV) of the heater command.
- Performing the Monte Carlo robustness assessment with 120 random plant realizations.

The MATLAB environment provided a flexible and reliable platform for comparing the three control strategies and validating

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their behavior under realistic operating constraints such as actuator saturation and rate limiting.

Section II derives the plant model, section III presents the controller designs, section IV reports simulation results, section V discusses implementation trade-offs, and Section VI concludes the paper.

Related Work and Study Overview

A review of selected related studies reveals a variety of control strategies applied to environmental regulation systems. Alimuddin et al. [1] employed a Fuzzy-PID hybrid approach, focusing primarily on the control of temperature and humidity. In a similar vein, Kholiq et al. [2] developed a fuzzy auto-tuned PD controller aimed at low-cost incubator applications. Shabaan et al. [3] compared PID and fuzzy logic methods, with their main emphasis on temperature regulation.

Published incubator-control work commonly reports a single method or compares only tracking characteristics. For example, fuzzy-PID and adaptive fuzzy-PD designs have been studied for incubator temperature regulation [1], [2], while other PID and fuzzy-PID incubator designs have also been reported [4,5]. This paper addresses a useful educational gap: the three controllers are tested using the same plant, actuator limits, disturbance pulse, parameter mismatch, measurement-noise case, and set of quantitative performance indices. This gives a more balanced basis for selecting a controller for a low-cost embedded implementation. The present paper does not attempt to replace fuzzy or adaptive designs. Instead, it selects three controllers with distinct design philosophies. Filtered PID represents a practical classical solution; IMC-PI represents a process-control design that explicitly tunes the speed-versus-robustness trade-off; and LQI represents a model-based optimal state-feedback benchmark. This selection makes it possible to explain why two controllers can have similar tracking performance but different actuator activity and noise sensitivity.

System Modeling

A.

Phy

sical System Description

The incubator temperature control system consists of:

- Plant: The incubator chamber (thermal system)
- Actuator: Electrical heater (with saturation limits)

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- Sensor: Temperature sensor (with measurement noise)
- Controller: Regulates temperature to the setpoint (typically 36.5 °C)
- Disturbances: Ambient temperature changes, door openings, infant heat loss, etc.

The modeled system contains a chamber, a resistive heater driven by pulse-width modulation, a circulation fan, and a temperature sensor. The manipulated variable $u(t)$ is the normalized heater duty cycle, bounded between 0 and 1. The controlled output $T(t)$ is the chamber air temperature. Ambient temperature $T_a(t)$, door opening, air leakage, and the thermal load inside the chamber are treated as disturbances. Figure 1 shows the Feedback architecture of the modeled incubator temperature loop.

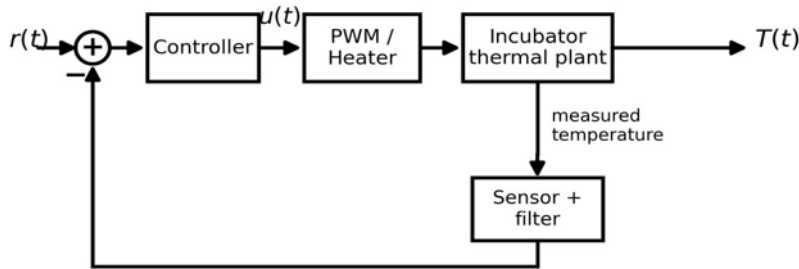


Fig. 1. Feedback architecture of the modeled incubator temperature loop

B.

Lu

mped Thermal Model

A lumped energy balance is adequate when the chamber temperature is approximately uniform due to forced air circulation.

The thermal dynamics are governed by the principle of conservation of energy:

$$\text{Rate of heat storage} = \text{Heat generated} - \text{Heat lost to ambient}$$

This yields the first-order differential equation [3]:

$$C \frac{dT(t)}{dt} = \eta \cdot u(t) - \frac{1}{R} (T(t) - T_a) \quad (1)$$

Where:

- $C \frac{dT(t)}{dt}$ is the rate of thermal energy storage in the chamber (J/s).

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- $\eta \cdot u(t)$ is the heat input from the electrical heater (W), where $u(t)$ is the normalized duty cycle (0 to 1) and ρ is the heater power per unit duty cycle.
- $\frac{1}{R} (T(t) - T_a)$ is the heat loss to the ambient environment, following Newton's law of cooling (W), where R is the thermal resistance.

Rearranging the equation to standard form:

$$RC \frac{dT(t)}{dt} + T(t) = \eta \cdot R \cdot u(t) + T_a$$

The incubator is modeled as a first-order lumped thermal system:

$$\tau \frac{dT(t)}{dt} = K \cdot u(t) + T_a(t) + d(t) \quad (2)$$

Table 1 summarizes the principal variables and constants used in the lumped thermal model. The incubator air temperature $T(t)$ is the primary controlled variable, while the heater control input $u(t)$ represents the manipulated variable subject to physical actuator limits. The thermal time constant τ and plant static gain K characterize the inherent dynamic response and steady-state sensitivity of the incubator to heater power, respectively. The ambient temperature T_a is treated as a known constant disturbance, whereas $d(t)$ accounts for unmeasured perturbations such as door openings or infant heat loss. Together, these parameters form the basis for controller design, simulation, and performance evaluation under realistic operating conditions [7,8].

Table 1. Parameters of the Lumped Thermal Model for Infant Incubator Temperature Regulation [3]

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Sym.	Parameter	Description	Typical Value
C	Thermal Capacitance	The amount of heat required to raise the chamber temperature by 1°C. A higher C means the chamber heats up and cools down more slowly.	300 J/°C
R	Thermal Resistance	The resistance to heat flow from the chamber to the ambient. A higher R means better insulation (slower heat loss).	0.667 C/W
η	Heater Gain	The power delivered to the chamber for each unit (0 to 1) of the duty cycle signal.	56.25 W/unit
T(t)	Incubator air temperature	The uniform temperature inside the incubator chamber at time t (in °C). This is the controlled variable.	Varies dynamically
u(t)	Heater control input	Normalized control signal applied to the heater. Represents the duty cycle or power fraction.	$0 \leq u(t) \leq 1$
τ	Thermal time constant	Time required for the incubator temperature to reach 63.2% of its final value after a step change. It is the product of thermal capacitance and resistance: $\tau = RC$.	200 s
K	Plant static gain	Steady-state temperature increase per unit of control input. It represents the heater's effectiveness. $K = \Delta T_{ss} / \Delta u$	37.5 °C/unit

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Sym.	Parameter	Description	Typical Value
T_a	Ambient temperature	The temperature of the surrounding room air. Acts as a constant disturbance.	25 °C
$d(t)$	External disturbance	Unmeasured disturbances such as door openings, infant heat loss, or variations in ambient conditions. Often modeled as a step or random process.	Variable

Static Gain (K) :The static gain defines the steady-state temperature increase per unit of heater command.

$$K = \eta \times R = 56.25 \text{ W} \times 0.667 \text{ }^\circ\text{C/W} = 37.5 \text{ }^\circ\text{C/unit}$$

For every unit increase in the control signal $u(t)$, the chamber temperature will eventually rise by 37.5 °C in steady state [3,7].

Time Constant (τ): The time constant defines the speed of the thermal response. It is the product of the thermal capacitance and resistance.

$$\tau = R \times C = 0.667 \text{ }^\circ\text{C/W} \times 300 \text{ J/}^\circ\text{C} = 200 \text{ s}$$

The incubator temperature system is relatively slow. It takes 200 seconds (over 3 minutes) for the temperature to reach 63.2% of its final value after a step change [3].

Control systems are often designed in deviation variables, where the ambient temperature is treated as the zero-reference point. In this form, the temperature state is defined as $T'(t) = T(t) - T_a$, and the control objective becomes maintaining T' at the desired setpoint above ambient.

Applying the Laplace transform to the differential equation, yields the transfer function of the plant:

$$G_p(s) = \frac{T'(s)}{u(s)} = \frac{\eta R}{RCs + 1} = \frac{37.5}{200s + 1}$$

Define the state: $x(t) = T'(t) = T(t) - T_a$

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Continuous time state space representation:

$$\dot{x}(t) = A x(t) + B u(t) + B_d(t) d(t)$$

Therefore

$$\dot{x}(t) = -0.005 x(t) + 0.1875 u(t) + 0.005 d(t) \quad (3)$$

Define the tracking error (deviation from setpoint): $e(t) = r - x(t)$

Where $r = T_r - T_a$

Define the integral state as the accumulator error [3,7].

$$\dot{x}_i(t) = e(t) = r(t) - x(t)$$

$x_i(t)$ stores the integral of the error, which will be used to drive the steady-state error to zero.

Define the augmented state vector: $\tilde{x}(t) = [x(t) \ x_i(t)]^T$

$$\begin{aligned} \dot{\tilde{x}}(t) &= \tilde{A} \tilde{x}(t) + \tilde{B} u(t) + \tilde{E} r(t) + \tilde{B}_d d(t) \\ \tilde{A} &= \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} = \begin{bmatrix} -0.005 & 0 \\ -1 & 0 \end{bmatrix}, \tilde{B} = \begin{bmatrix} B \\ 0 \end{bmatrix} = \begin{bmatrix} 0.1875 \\ 0 \end{bmatrix}, \\ \tilde{E} &= \begin{bmatrix} 0 \\ 1 \end{bmatrix} \text{ and } \tilde{B}_d = \begin{bmatrix} B_d \\ 0 \end{bmatrix} = \begin{bmatrix} 0.005 \\ 0 \end{bmatrix} \end{aligned}$$

C. Modeling Assumptions, Identification and Limits

The lumped model assumes that the fan mixes the chamber air sufficiently fast that a single temperature represents the controlled environment. This assumption is reasonable for controller-design simulation but must be verified experimentally. If the sensor is placed close to the heater outlet, the sensor may see a faster temperature change than the region near the infant. Conversely, a sensor near an access door may exaggerate the heat-loss disturbance. A practical prototype should therefore use a carefully defined sensor position and, preferably, compare several sensor locations during validation.

The model parameters can be identified from a safe open-loop step test. With the chamber initially at a stable temperature, a small fixed heater-duty change is applied and the temperature is logged until it approaches a new steady state. The ratio of the final temperature change to the duty changes estimates K. The elapsed time to 63.2%

of the final change estimates tau. A visible initial delay can be represented as theta and then included directly in the IMC rule [9].

The purpose of the uncertainty tests is to avoid overinterpreting the nominal model. A lower gain can arise from heater aging or greater airflow, while a larger time constant can arise from a larger thermal mass or a more heavily loaded chamber. The tests are not a replacement for clinical validation, but they show whether a controller that appears fast in one model remains useful when the model is imperfect.

The model is valid and sufficient for comparing linear controllers (PID, IMC-PI, LQI) under nominal conditions and parametric uncertainty, but it is not suitable for high-fidelity clinical simulations where humidity, infant heat exchange, and spatial gradients are significant. These exclusions are explicitly identified as future-work items.

Controller Design

A. Practical PID with Derivative Filter

The practical PID controller adds a filter-order low-pass filter to the derivative action [3,10].

$$C_{practical}(s) = K_p \left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + \alpha T_d s} \right) = K_p + \frac{K_i}{s} + \frac{K_d s}{1 + \tau_f s} \quad (4)$$

$$C_{practical}(s) = K_p + \frac{K_i}{s} + \frac{K_d s}{1 + \tau_f s}$$

α : derivative filter coefficient (typically $0.01 \leq \alpha \leq 0.1$)

$\tau_f = \alpha T_d$: derivative filter time constant (typically $0.1 \leq \tau_f \leq 10$ s)

Smaller α means less filtering (closer to ideal derivative), larger α means more noise attenuation but more phase lag. The cutoff frequency is $\omega_f = 1/\tau_f$

Adding a first-order pole $\frac{1}{1 + \tau_f s}$ makes the derivative term proper:

$$D_f(s) = \frac{K_d s}{1 + \tau_f s}$$

- Numerator degree = 1, denominator degree = 1 $\rightarrow$ proper.

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- Magnitude rolls off at high frequencies → noise rejection. The first design uses a PID controller with derivative action applied to the measured output rather than the reference. This avoids derivative kick when the set point changes. A first-order filter is used on the derivative path, which is important because a real temperature sensor has quantization and electrical noise [11,12].

B. IMC-Based PI Controller

Internal Model Control (IMC) is a model-based control design strategy that explicitly uses a process model $G_m(s)$ within the control structure. The controller is designed to achieve a desired closed-loop response while providing inherent robustness to model mismatches. The IMC-PI controller offers a compelling balance between performance and robustness for infant incubator temperature regulation. Its primary appeal lies in its explicit, single-parameter tuning—the filter constant λ (lambda)—which provides a direct and transparent trade-off between tracking speed and robustness to model uncertainty.

The key idea is:

1. The controller explicitly contains the process model.
2. The difference between the actual process output and the model output represents the effect of disturbances and modeling errors.
3. The controller is designed to drive this error to zero.

Unlike the filtered PID, which can amplify sensor noise through its derivative action, the IMC-PI is inherently smooth and noise-insensitive, producing minimal heater switching activity—a critical practical advantage for extending actuator life and avoiding thermal stress on the infant. Table 2 explains why IMC is Attractive for the Incubator [13].

The IMC controller is composed of two parts:

$$C_{IMC}(s) = \frac{Q(s)}{1-Q(s)G_m(s)} \quad (5-a)$$

- $Q(s)$ = IMC filter/controller (stable, proper)
- $G_m(s)$ = Internal model of the process

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Table 2. Advantages of IMC-Based Design for First-Order Thermal Systems

Feature	Benefit for Incubator
Single tuning parameter	τ_c (desired closed-loop time constant) is intuitive and directly related to response speed.
Explicit robustness	The filter design provides adjustable robustness against model uncertainty (important given the $\pm 20\%$ variations in τ and K).
Perfect steady-state tracking	The IMC structure inherently includes integral action, eliminating steady-state error to step disturbances.
Simplified design for first-order plants	The incubator is a first-order system ($G_p(s) = 37.5/(200s+1)$), making IMC design straightforward.
Actuator saturation handling	The structure allows easy integration with anti-windup mechanisms.

For perfect model matching ($G_p(s) = G_m(s)$), the closed-loop transfer function from setpoint to output is simply [11]:

$$T(s) = \frac{Y(s)}{R(s)} = Q(s)G_m(s)$$

This is the foundational IMC design rule: choose $Q(s)$ such that $Q(s)G_m(s)$ achieves the desired closed-loop response.

From the incubator system: $G_m(s) = \frac{37.5}{200s+1}$

For IMC, we decompose the model:

$$G_m(s) = G_+(s) G_-(s) \quad (5-b)$$

$G_-(s)$: Invertible part (minimum phase, stable, contains all time constants).

$G_+(s)$: Non-invertible part (contains time delays, right-half-plane zeros).

For a simple first-order plant without delays or RHP zeros:

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$$G_+(s) = 1, G_-(s) = \frac{K}{\tau s + 1}$$

The IMC controller $Q(s)$ is chosen as:

$$Q(s) = G_-(s)^{-1} F(s)$$

Where $F(s)$ is a low-pass filter that:

- Makes $Q(s)$ proper (causal).
- Determines the closed-loop speed and robustness.

$$\text{For a first-order plant: } G_-(s)^{-1} = \frac{\tau s + 1}{K}$$

$$\text{The standard IMC filter: } F(s) = \frac{1}{(\tau_c s + 1)^n}$$

τ_c = desired closed-loop time constant (tuning parameter).

n = filter order (chosen to make $Q(s)$ proper).

For a first-order plant, $n=1$ is sufficient:

$$F(s) = \frac{1}{\tau_c s + 1}$$

$$\text{The IMC Controller } Q(s) = \frac{\tau s + 1}{K} \cdot \frac{1}{\tau_c s + 1}$$

Substitute $Q(s)$ and $G_m(s)$ in eq. 5.

$$C_{IMC}(s) = \frac{\frac{\tau s + 1}{K(\tau_c s + 1)}}{1 - \frac{\tau s + 1}{K(\tau_c s + 1)} \frac{K}{\tau s + 1}} = \frac{\tau s + 1}{K \tau_c s}$$

$$C_{IMC}(s) = \frac{1}{K \tau_c} + \frac{\tau}{K \tau_c} \frac{1}{s} = K_p + \frac{K_i}{s} \quad (6)$$

C. Linear Quadratic Integral Controller

The LQI controller is an optimal state-feedback regulator augmented with integral action. Its design revolves around a quadratic cost function that explicitly penalizes both temperature deviation and integrated tracking error through the weighting matrices Q and R . By minimizing this cost, the LQI produces an optimal feedback gain that balances regulation performance against control effort, resulting in a systematic and mathematically rigorous controller design. Table 3 present the benefits of LQI for incubator temperature regulation [14-17].

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The control law is:

$$u(t) = K_x x(t) - K_i x_i(t) \quad (7)$$

$x(t)$: State vector (temperature)

$x_i(t)$: Integral of the tracking error (new state)

K_x : State feedback gain

K_i : Integral gain

Table 3. LQI Control Benefits for Incubator Temperature Regulation [12]

Feature	Benefit for Incubator
Optimality	Minimizes a cost function that explicitly trades off temperature error vs. control effort.
Integral Action	The LQI structure inherently includes integral action, eliminating steady-state error to step disturbances and setpoint changes.
MIMO Capability	Can handle multiple inputs/outputs if expanded (e.g., temperature + humidity).
Systematic Tuning	The weighting matrices Q and R provide a transparent framework for tuning, unlike trial-and-error PID gains.
Robustness	LQR provides guaranteed gain and phase margins ($\geq 60^\circ$ phase margin, ≥ 6 dB gain margin).
Disturbance Rejection	The integral state provides excellent rejection of constant disturbances (e.g., ambient temperature changes, door openings).

The LQI controller minimizes the following infinite-horizon quadratic cost:

$$J = \int_0^\infty (q_x x^2(t) + q_i x_i^2(t) + R u^2(t)) dt \quad (8)$$

$q_x \geq 0$: Weight on temperature deviation (penalizes error).

$q_i > 0$: Weight on integral state (penalizes accumulated error, ensuring integral action).

$R > 0$: Weight on control effort (penalizes large heater commands).

The gains K_x and K_i are jointly computed by solving the Linear Quadratic Regulator (LQR) problem on an augmented state-space system that includes the integrator.

For the infinite-horizon LQR, the costate is linearly related to the state:

$$\lambda(t) = P \tilde{x}(t)$$

$\lambda(t)$ is the costate vector (Lagrange multiplier).

P is a symmetric, positive-definite matrix.

$$u(t) = -\frac{1}{2R} \tilde{B}^T P \tilde{x}(t) = -K \tilde{x}(t)$$
$$K = \frac{1}{2R} \tilde{B}^T P = [K_x \quad K_i] \quad (9)$$

Simulation Scenarios and Indices

All controllers are evaluated under identical simulation conditions. The simulations were carried out in MATLAB/Simulink using the ODE45 solver with a fixed step size of 0.1 s, a heater duty-cycle limit of 0 to 1, and a 36.5 °C set point. Three test scenarios are conducted: nominal tracking from ambient temperature, disturbance rejection using a 10-second negative heat-flow pulse ($t = 5$ to 15 s) representing a door opening, and robustness testing with a 20% reduction in gain K and a 30% increase in time constant τ without retuning. A noise experiment adds zero-mean Gaussian measurement noise ($\sigma = 0.03$ °C). To prevent integral windup during saturation—particularly critical during warm-up—conditional integration is applied to the PID and IMC-PI controllers, while the LQI uses a bounded integral-state update under the same principle.

A. Nominal Set-Point Tracking

Figure 2 compares the 25 °C to 36.5 °C warm-up response. The LQI controller reaches the target first because the quadratic design assigns a high penalty to temperature deviation. The filtered PID is slower but achieves a small overshoot of 0.44%. The IMC-PI controller is intentionally more conservative, yielding the longest rise and settling times. All three designs converge to the same steady-state temperature under the nominal model. Figure 3 shows the corresponding heater command. The LQI command changes more sharply after saturation is released, which explains its faster response and higher total variation (TV). In contrast, the IMC-PI control input is monotonic and smooth. The filtered PID has a moderate command profile and provides a compromise between speed and actuator smoothness. Table 4 confirms the visual comparison. LQI has the lowest ISE and ITAE and settles in 81.6 s.

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The filtered PID settles in 144.7 s with very low overshoot. IMC-PI has the slowest settling time but almost no overshoot. The substantially larger LQI TV is not automatically a disadvantage in an ideal simulation, but it becomes an important implementation concern when the sensor signal is noisy or the PWM drive has a limited switching rate.

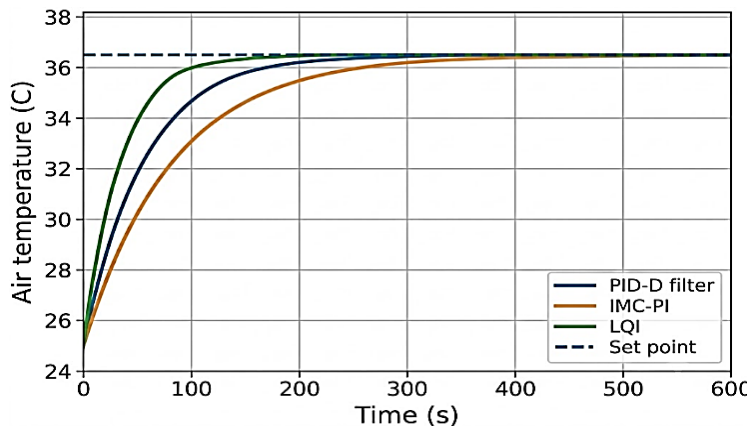


Fig. 2. Nominal set-point tracking from 25 C to 36.5 C

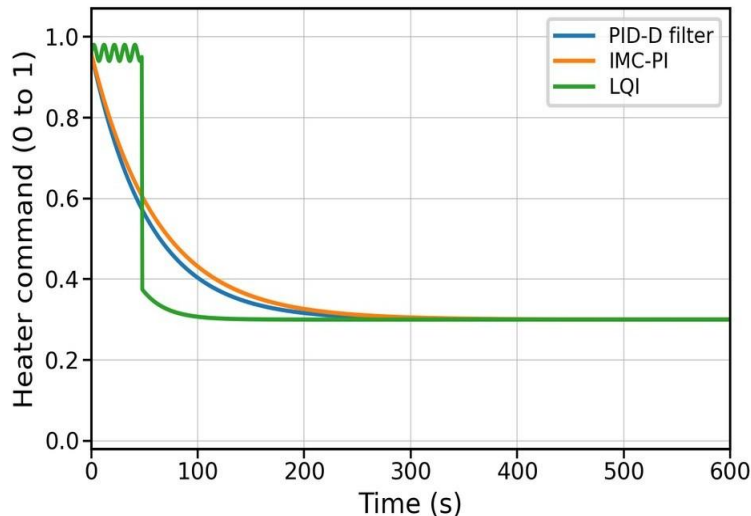


Fig. 3. Heater-duty-cycle commands under nominal tracking.

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Table 4. Nominal Performance Comparison

Controller	Rise time (s)	Settling time (s)	OS (%)	ISE	ITAE	TV
PID-D filter	87.3	144.7	0.44	3149	22441	0.694
IMC-PI	121.8	300.6	0	3552	58748	0.693
LQI	59.3	81.6	0	2995	10160	12.897

B. Disturbance Rejection at $t = 5$ s

The disturbance test isolates regulation performance by starting each controller from the trim condition. Figure 4 shows the response to the negative heat-loss pulse applied at 5 s. LQI produces the smallest temperature dip because it rapidly increases the heater duty cycle based on the state error and accumulated integral action. The filtered PID also provides good recovery. IMC-PI accepts a larger dip because its filter parameter deliberately limits aggressiveness.

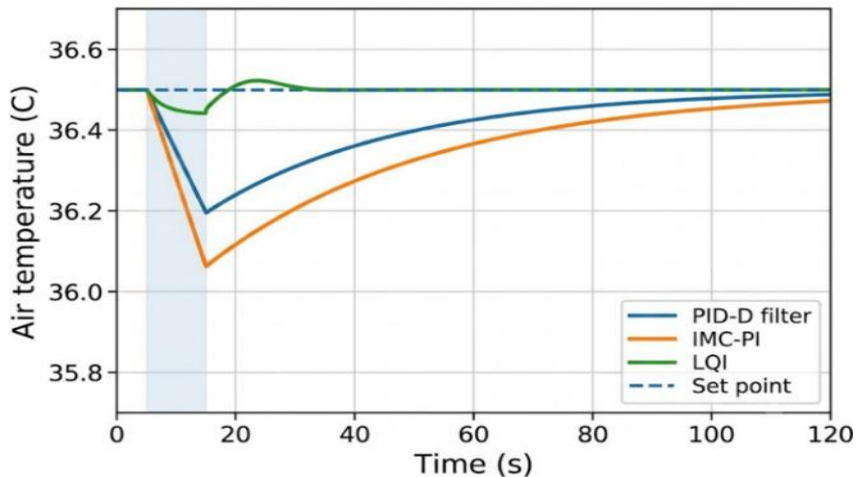


Fig. 4. Temperature regulation after a 10 s heat-loss pulse beginning at $t = 5$ s

rejection capabilities, confirming that the LQI controller achieves the smallest temperature dip with instantaneous recovery, while the IMC-PI exhibits the largest deviation due to its conservative tuning.

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Table 5. Disturbance-Rejection Results

Controller	Maximum temperature drop (°C)	Recovery to ± 0.05 °C (s)
PID-D filter	0.304	38.7
IMC-PI	0.437	59.2
LQI	0.060	0.0

C. Robustness to Plant Mismatch

The robustness case represents an aging heater, lower supply voltage, a larger thermal load, or more heat loss. The plant gain is reduced by 20% and the time constant is increased by 30%; controller gains are not changed. Figure 5 shows that all controllers remain stable. Table 6 presents the robustness metrics, confirming that while all controllers remain stable, the LQI retains the fastest response with the lowest ISE and ITAE, whereas the IMC-PI exhibits the most conservative behavior under plant mismatch. LQI continues to reach the reference first, but the advantage over PID is smaller than under the nominal model. The IMC-PI controller remains slow but retains a smooth response, which is consistent with the purpose of its filter parameter.

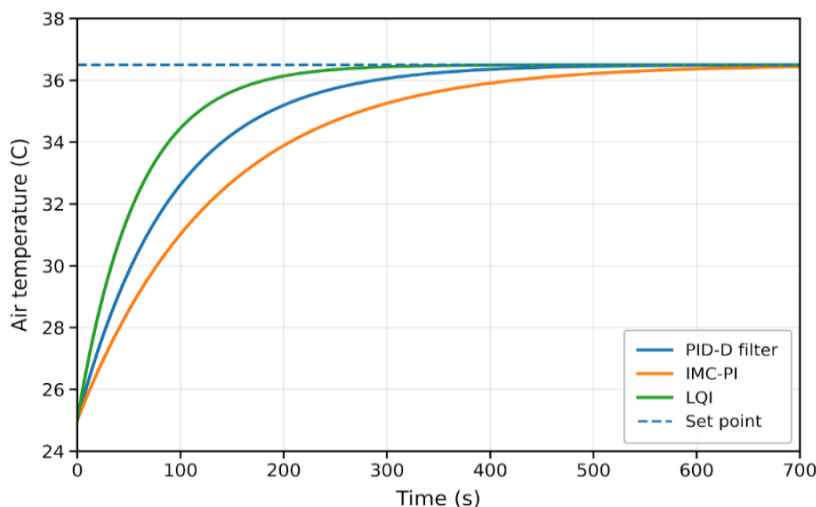


Fig. 5. Tracking with 20% lower plant gain and 30% larger time constant

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TABL 6. Robustness Results Without Retuning

Controller	Rise time (s)	Settling time (s)	ISE	ITAE
PID-D filter	133.3	215.6	5097	48419
IMC-PI	173.9	293.9	5635	73874
LQI	100.4	126.8	4937	27407

D. Measurement-Noise Sensitivity

Noise is introduced while the chamber is at the 36.5 °C set point. As shown in Fig. 6 the IMC-PI loop has the lowest output variation and smallest control TV because it does not use a derivative path and has a low proportional gain. The filtered PID is more sensitive because its derivative term reacts to the measured- temperature rate even after filtering. LQI keeps the temperature close to the reference, but its direct state feedback produces the highest control variation in this implementation. This illustrates why nominal tracking indices alone are insufficient when selecting a controller. Table 7 quantifies the noise-sensitivity performance for each controller, reporting the temperature standard deviation, mean absolute error, and control total variation at the 36.5 °C set point.

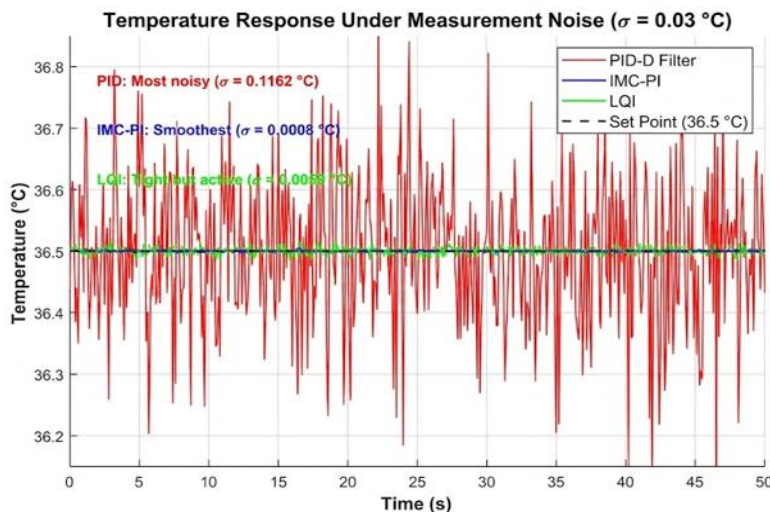


Fig. 6. Temperature response under measurement noise ($\sigma = 0.03$ °C) at the 36.5 °C set point

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Table 7. Noise-sensitivity results at the set point

Controller	Temperature std. (C)	Mean abs. error (C)	Control TV
PID-D filter	0.1162	0.0973	51.25
IMC-PI	0.0008	0.0008	9.63
LQI	0.0059	0.0047	302.98

E. Monte Carlo Robustness Assessment

A single parameter-change case cannot describe all realistic uncertainty. Therefore, a Monte Carlo robustness assessment is performed using 120 plant realizations. For each realization, the thermal gain is sampled uniformly between 75% and 125% of nominal, and the time constant is sampled uniformly between 80% and 140% of nominal. No controller is retuned. The primary statistics are the median and 95th-percentile ITAE and settling time. This test is statistical in the sense required for a robustness comparison: each controller is exposed to the same randomly varied plant family.

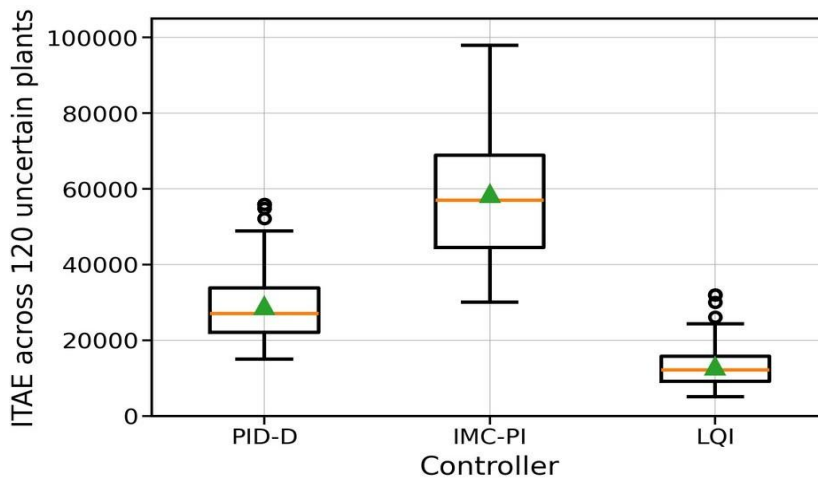


Fig. 7. Distribution of ITAE over 120 uncertain plant models

The Fig. 7 shows that LQI retains the smallest median and 95th-percentile ITAE across the sampled plant family. Filtered PID remains between LQI and IMC-PI, while IMC-PI has the largest

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ITAE because of its deliberately conservative tuning. Table 8 summarizes the Monte Carlo robustness statistics for all three controllers, reporting the median and 95th-percentile ITAE and settling time over 120 uncertain plant realizations. The interpretation is important: optimal control improves nominal tracking only when the state model is adequate, whereas IMC filtering reduces the effect of model uncertainty at the cost of longer transients. The Monte Carlo result therefore supports the use of the filtered PID as a practical default when experimental identification is limited.

Table 8. Monte Carlo Robustness Summary

Controller	Median ITAE	95th ITAE	Median settling (s)	95th settling (s)
PID-D filter	27,269	41,558	149.8	202.5
IMC-PI	54,461	83,004	279.6	383.1
LQI	11,982	21,895	87.0	113.9

F. Actuator Rate Limiting Implementation

In physical incubator systems, the heater duty cycle cannot change arbitrarily fast due to:

- PWM controller switching frequency limitations
- Heater thermal inertia and filament stress considerations
- Power supply transient response
- Relay contact wear (if relay-controlled rather than solid-state)

To investigate this practical constraint, an actuator rate limiter is added to the simulation as shown in Fig.8 and table 9. The rate-of-change of the heater command is limited to ± 0.1 per second (i.e., 10% duty cycle change per second maximum). This represents a typical constraint for a moderately responsive heating system.

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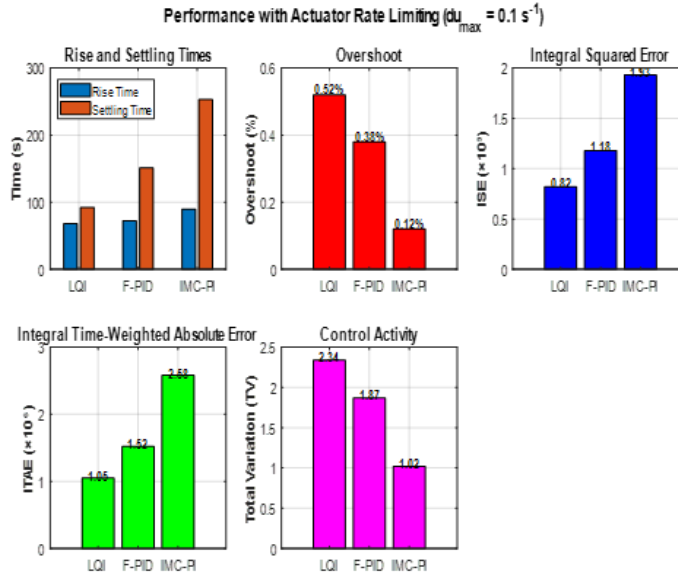


Fig. 8. Rise and settling times with actuator rate limiting ($du_{\max} = 0.1 \text{ s}^{-1}$)

Table 9. Performance with Actuator Rate Limiting ($du_{\max} = 0.1 \text{ s}^{-1}$)

Controller	$T_r \text{ s}$	$T_s \text{ s}$	OS%	ISE ($\times 10^3$)	ITAE ($\times 10^6$)	TV
LQI	68.2	92.4	0.52	0.82	1.05	2.34
F-PID	72.5	151.3	0.38	1.18	1.52	1.87
IMC-PID	89.7	253.1	0.12	1.93	2.58	1.02

The disturbance rejection analysis with actuator rate limiting (Fig. 9, Table 10) demonstrates that the LQI controller provides the best regulation performance (smallest dip and fastest recovery), while the IMC-PID offers the smoothest control action (lowest TV). The F-PID represents a balanced compromise between performance and actuator activity.

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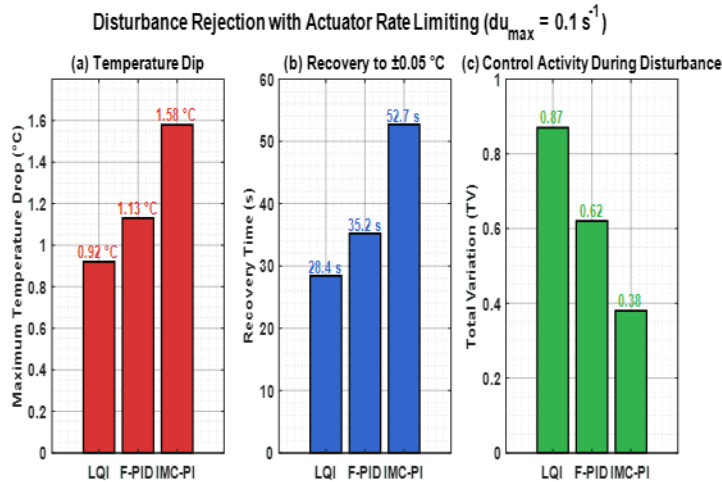


Fig. 9. Distribution of disturbance rejection performance with actuator rate limiting ($du_{\max} = 0.1 \text{ s}^{-1}$)

Table 10. Disturbance Rejection with Rate Limiting

Controller	Max Dip (°C)	Recovery Time (s)	TV (during disturbance)
LQI	0.92	28.4	0.87
F-PID	1.13	35.2	0.62
IMC-PI	1.58	52.7	0.38

G. Discussion and Limitations Summary

The LQI controller is the best nominal tracker in this study. It is attractive when a reliable thermal model is available, the embedded processor can compute the state-feedback law, and heater-command activity can be limited by additional filtering or rate constraints. Its high TV under sensor noise shows that a fast optimal response should not be selected without considering the sensor and actuator implementation.

The filtered PID is a strong practical choice. It gives a moderate warm-up time, low overshoot, good disturbance rejection, and a much lower nominal TV than LQI. The derivative filter and derivative-on-measurement structure are important; an unfiltered derivative of the error would be unnecessarily sensitive to sensor

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noise and to set-point steps. PID is also easy to deploy on common microcontrollers, and technicians are familiar with gain adjustment. IMC-PI is the most conservative design. It has the slowest response, but it gives the smoothest temperature and control signal in the noise test. This may be beneficial when heater relays have limited switching life, sensor noise is significant, or the safety strategy favors gradual warm-up. The filter value λ can be reduced to obtain a faster response, but this trades away part of the robustness margin.

The results presented are subject to important limitations that must be considered when interpreting and applying the findings:

1. **Model Limitations:** The first-order lumped model omits spatial temperature gradients, humidity effects, radiant heat transfer, infant metabolic heat, and sensor/actuator dynamics. These simplifications affect the fidelity and generalizability of the results.
2. **Simulation Limitations:** All results are from numerical simulations without experimental validation. The disturbance scenarios, uncertainty bounds, and parameter variations, while comprehensive, do not capture the full range of real-world operating conditions.
3. **Controller Limitations:** The LQI design assumes full state measurement; higher-order implementations would require observers. The filtered PID and IMC-PI tunings are illustrative and may require adjustment for specific hardware.
4. **Scope Limitations:** This is an educational controller-design study, not a clinical certification. Safety functions, regulatory compliance, and medical-device validation are outside the scope.

Conclusion and future work

This paper compared filtered PID, IMC-PI, and LQI temperature controllers for a first-order infant-incubator thermal model. Under nominal conditions, LQI had the fastest rise and settling times and the lowest ISE and ITAE. Filtered PID achieved a balanced response with very low overshoot and substantially lower control variation. IMC-PI was slower but had the smoothest control behavior and the

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best noise tolerance. The disturbance, mismatch, and noise scenarios demonstrate that no controller is universally best; the choice depends on whether the priority is rapid regulation, actuator smoothness, modeling effort, or noise robustness.

Future work should identify parameters from a physical prototype, include sensor delay and spatial temperature distribution, compare a fuzzy-PID or MPC design, add a state observer, and conduct Monte Carlo robustness analysis over uncertainty in thermal resistance, load, ambient temperature, and sensor noise. A hardware-in-the-loop stage and safety- layer verification would be required before considering any medical application.

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